Oscillations harmoniques: système masse-ressort et circuit L-C résolution

jeudi, 31 décembre 2020 16:10

Masse - ressort

$$\boxed{\ddot{z} = -\frac{k}{m} z}$$

Conditions initiales:

$t = 0$

$z(0) = x(0) - l_0 = A$

$\dot{z}(0) = 0$

Circuit L-C série

$$\boxed{\ddot{q} = -\frac{1}{LC} q}$$

Conditions initiales:

$t = 0$

$q(0) = Q_0$

$\dot{q}(0) = 0$

· Forme générale : $\ddot{y} = -\alpha^2 y$

On remarque :

$\sin(x)' = \cos(x)$

$\sin(x)'' = \cos(x)' = -\sin(x)$

On essaye $y(t) = B \sin(\omega t + \varphi)$

$\dot{y} = B\omega \cos(\omega t + \varphi)$

$\ddot{y} = -B\omega^2 \sin(\omega t + \varphi)$

$\Rightarrow$ En $t = 0$: $y(0) = B \sin(\varphi)$ $\rightarrow$ A (mécanique) $\Rightarrow B = A \,;\, \varphi = \frac{\pi}{2}$

$\rightarrow$ Q_0 (électro...) $\Rightarrow B = Q_0 \,;\, \varphi = \frac{\pi}{2}$

$-\cancel{B}\omega^2 \cancel{\sin(\omega t + \varphi)} = -\alpha^2 \cancel{B} \cancel{\sin(\omega t + \varphi)}$

$\omega = \alpha$ $\rightarrow$ $\alpha = \sqrt{\frac{k}{m}}$

$\rightarrow$ $\alpha = \sqrt{\frac{1}{LC}}$

$\Rightarrow$

$$\boxed{z(t) = A \sin\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right) = x(t) - l_0}$$

$$\boxed{q(t) = Q_0 \sin\left(\sqrt{\frac{1}{LC}} \cdot t + \frac{\pi}{2}\right)}$$