Corrigé exercices moh

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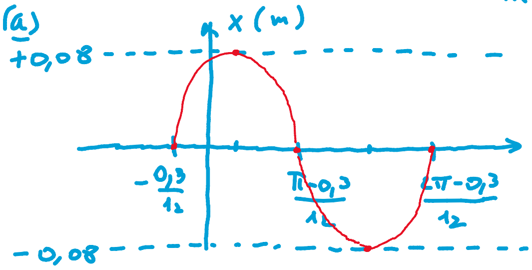
① . $x(t) = 0{,}080 \cdot \sin(\overbrace{12 \cdot t + 0.30}^{\text{en radians !}})$

(a)




dessin pour une période

• $x(t) = 0$ si $12 \cdot t + 0{,}3 = n \cdot \pi$

$\Rightarrow t = \frac{n\pi - 0{,}3}{12} = \left\{ \dots ; -\frac{0{,}3}{12} ; \frac{\pi - 0{,}3}{12} ; \frac{2\pi - 0{,}3}{12} ; \dots \right\}$

• les extrêmes de $x(t)$ sont définis par $12 \cdot t + 0{,}3 = (2m+1)\frac{\pi}{2}$

$\Rightarrow t = \frac{(2m+1)\pi/2 - 0{,}3}{12} = \left\{ \dots ; \frac{\pi}{24} - \frac{0{,}3}{12} ; \frac{3\pi}{24} - \frac{0{,}3}{12} ; \dots \right\}$

$\Rightarrow$ pour $t = \frac{\pi}{24} - \frac{0{,}3}{12}$ , $x(t) = 0{,}080$

(b). $x(0{,}60) = 0{,}080 \cdot \sin(12 \cdot 0{,}6 + 0{,}3) \cong \underline{\underline{0{,}075 \text{ m}}}$

$v(0{,}60) = \dot{x}(0{,}60) = \frac{d}{dt} x(0{,}60) = 12 \cdot 0{,}080 \cdot \cos(12 \cdot 0{,}6 + 0{,}3) \cong \underline{\underline{0{,}33 \text{ m/s}}}$

$a(0{,}60) = \ddot{x}(0{,}60) = \frac{d}{dt} v(0{,}60) = -12^2 \cdot 0{,}080 \cdot \sin(12 \cdot 0{,}6 + 0{,}3) \cong \underline{-11 \text{ m/s}^2}$

(c). a pour $x = -0{,}050$ m ? On sait que $\ddot{x} = -12^2 x$ (cf question b !) $\Rightarrow \ddot{x} = -12^2 \cdot (-0{,}050) \cong \underline{\underline{7{,}2 \text{ m/s}^2}}$

② . $x(t) = 0{,}050 \sin(10 \cdot t + \frac{\pi}{2})$

(a). $E_{méc} = E_{cin} + E_{pot} = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$v = \dot{x} = \frac{dx}{dt} = 0{,}050 \cdot 10 \cdot \cos(10 \cdot t + \pi/2)$

$x(t = \frac{\pi}{15}) = 0{,}050 \sin\left(\frac{2\pi}{3} + \frac{\pi}{2}\right) = 0{,}050 \sin\left(\frac{7\pi}{6}\right) \Rightarrow E_{pot} = \frac{1}{2} k x^2 = \frac{1}{2} \cdot 200 \cdot 0{,}05^2 \cdot \sin^2\left(\frac{7\pi}{6}\right) = \underline{0{,}063 \text{ J}}$

$v(t = \frac{\pi}{15}) = 0{,}050 \cdot 10 \cos\left(\frac{7\pi}{6}\right) = 0{,}5 \cos\left(\frac{7\pi}{6}\right) \Rightarrow E_{cin} = \frac{1}{2} m v^2 = \frac{1}{2} \cdot 2 \cdot 0{,}5^2 \cos^2\left(\frac{7\pi}{6}\right) \cong \underline{0{,}19 \text{ J}}$

$\Rightarrow E_{méc}(t = \frac{\pi}{15}) = \underline{0{,}25 \text{ J}}$ (en additionnant les valeurs avec tous les chiffres de Ecin et Epot)

(b). On peut (!) utiliser le résultat de (a). $E_{cin}(x = \frac{0{,}050}{2}) = E_{méc} - E_{pot}(x = \frac{0{,}050}{2})$

$\frac{1}{2} m v^2 = 0{,}25 - \frac{1}{2} \cdot 200 \cdot \left(\frac{0{,}05}{2}\right)^2 = 0{,}1875 \text{ J}$

$\Rightarrow v = \sqrt{0{,}1875} \cong \underline{\underline{0{,}43 \text{ m/s}}}$

(c). $\frac{1}{2} m v^2 = \frac{1}{2} k x^2 = \frac{E_{méc}}{2} \Rightarrow x^2 = \frac{E_{méc}}{k} \Rightarrow x = \pm\sqrt{\frac{E_{méc}}{k}}$

Hors, on sait que $E_{méc} = \frac{1}{2} k A^2 \Rightarrow x = \pm\sqrt{\frac{1}{2} A^2} = \pm \underline{A \frac{\sqrt{2}}{2}}$
(pour les MOH)

en $t=0$, on veut que $x=0{,}050$

l'unité !

③. (a). $x(t) = 0{,}050 \sin\left(10 \cdot t + \frac{\pi}{2}\right) \quad (m)$

(b). $\ddot{x} = -10^2 x = -100 \cdot \frac{A}{2} = -50 \cdot A = -50 \cdot 0{,}05 = \underline{-2{,}5} \ m/s^2$

(c). $|F| = |k\, x(t = \frac{\pi}{15})| = |200 \cdot 0{,}050 \cdot \sin\left(\frac{7\pi}{6}\right)| \cong \underline{5{,}0 N}$

(d). On pose $x = -\frac{A}{2} = -0{,}025 = 0{,}050 \sin\left(10t + \frac{\pi}{2}\right)$

$\Rightarrow \sin\left(10 \cdot t + \frac{\pi}{2}\right) = -\frac{0{,}025}{0{,}050} = -0{,}50 \Rightarrow 10t + \frac{\pi}{2} = \arcsin(-0{,}5) = -\frac{\pi}{6}$

les angles suivants qui donnent un sinus de $-0{,}5$ sont : $\pi - (-\pi/6) = \frac{7\pi}{6}$

$-\frac{\pi}{6} + 2\pi = \frac{11\pi}{6}$

$\frac{7\pi}{6} + 2\pi = \frac{19\pi}{6}$

Pour obtenir un $t > 0$, on prend les angles $\left\{\frac{7\pi}{6} ; \frac{11\pi}{6} ; \frac{19\pi}{6}\right\}$

D'où : $t_1 = \left(\frac{7\pi}{6} - \frac{\pi}{2}\right)\frac{1}{10} \cong \underline{0{,}209 s}$ $\quad t_2 = \left(\frac{11\pi}{6} - \frac{\pi}{2}\right) \cdot \frac{1}{10} \cong \underline{0{,}419 s}$

$t_3 = \left(\frac{19\pi}{6} - \frac{\pi}{2}\right)\frac{1}{10} \cong \underline{0{,}838 s}$

④. (a) $\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^2\right) + \frac{d}{dt}\left(\frac{1}{2} k x^2\right) = \frac{1}{2} m \frac{d}{dt}(v^2) + \frac{1}{2} k \frac{d}{dt}(x^2) = m \dot{x} \ddot{x} + k x \dot{x} = 0$

$\begin{cases} \frac{d}{dt}(v^2) = 2v \frac{dv}{dt} = 2v \cdot \ddot{x} = 2\dot{x}\ddot{x} \\ \frac{d}{dt}(x^2) = 2x \frac{dx}{dt} = 2x \cdot \dot{x} \end{cases}$

$\Rightarrow m\ddot{x} = -kx \quad$ ou $\quad \ddot{x} = -\frac{k}{m} x$

(b). $\frac{dE}{dx} = \frac{d}{dx}\left(\frac{1}{2} m v^2\right) + \frac{d}{dx}\left(\frac{1}{2} k x^2\right) = \frac{1}{2} m \frac{d}{dx}(v^2) + \frac{1}{2} k \frac{d}{dx}(x^2) = m \dot{x} \ddot{x} \cdot \frac{1}{\dot{x}} + kx = 0$

$\begin{cases} \frac{d}{dx}(v^2) = 2v \frac{dv}{dx} = 2v \cdot \frac{dv}{dt} \cdot \frac{dt}{dx} = 2\dot{x}\ddot{x}\frac{dt}{dx} = 2\dot{x}\ddot{x}\frac{1}{\dot{x}} \\ \frac{d}{dx}(x^2) = 2x \end{cases}$

$\Rightarrow m\ddot{x} = -kx \quad$ ou $\quad \ddot{x} = -\frac{k}{m} x$

5. $y(t) = y_0 \cos\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right)$ (m)

(a). $\dot{y} = -y_0 \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right)$ ; $\ddot{y} = -y_0 \frac{k}{m} \cos\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right) = -\frac{k}{m} y_0 \cos\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right) = -\frac{k}{m} y$

(b). $V = \dot{y}$ et $V_{max} = y_0 \sqrt{\frac{k}{m}}$ , atteinte lorsque $\sin\left(\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2}\right) = \pm 1$ ← On choisit la valeur maximale de $|V|$

$$\Rightarrow \sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2} = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \sqrt{\frac{k}{m}} \cdot t = 2n\frac{\pi}{2} = n \cdot \pi$$

$$\Rightarrow \underline{\underline{t = \sqrt{\frac{m}{k}} \cdot n\pi}}$$

(c). $E_{pot} = \frac{1}{2} k y^2 \Rightarrow E_{pot\,max} = \underline{\underline{\frac{1}{2} k y_0^2}}$

(d). On a vu en (b) que si $\underline{\underline{t = \sqrt{\frac{m}{k}}}} \cdot \pi$ alors $\sqrt{\frac{k}{m}} \cdot t + \frac{\pi}{2} = \frac{3\pi}{2} \Rightarrow y\left(\sqrt{\frac{m}{k}} \cdot \pi\right) = \underline{\underline{0}}$