Chute verticale avec frottement et charge condensateur II

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Chute : $\dot v = g - \frac{\lambda}{m} v$

Condensateur : $\dot u_c = \frac{U}{RC} - \frac{1}{RC} u_c$

$\Rightarrow \boxed{\dot x = A + B \cdot x}$

$A = g \quad ; \quad B = -\frac{\lambda}{m}$

$A = \frac{U}{RC} \quad ; \quad B = -\frac{1}{RC}$

$$\boxed{x(t) = \frac{1}{B}\left(A e^{B \cdot t} - A\right)}$$

$$v(t) = -\frac{m}{\lambda}\left(g e^{-\frac{\lambda}{m} t} - g\right)$$

$$v(t) = \frac{mg}{\lambda}\left(1 - e^{-\frac{\lambda}{m} t}\right)$$

$$u_c(t) = -RC\left(\frac{U}{RC} e^{-\frac{t}{RC}} - \frac{U}{RC}\right)$$

$$u_c(t) = U\left(1 - e^{-\frac{1}{RC} t}\right)$$

idée : $\int \frac{f'(x)}{f(x)}\, dx = \ln(f(x)) + c \quad // \quad \frac{1}{B} \cdot \frac{B \dot x}{A + B x} = 1$

$f'(x) \mapsto B \dot x$

$f(x) \mapsto A + Bx$

$x \mapsto t$

$dx \mapsto dt$

$$\Rightarrow \frac{1}{B} \cdot \frac{B \dot x}{A + B x} = 1 \xrightarrow{\int_{t=0}^{t} dt'} \frac{1}{B} \int_{t=0}^{t} \frac{B \dot x}{A + B x}\, dt' = \int_{t=0}^{t} dt' \Rightarrow \frac{1}{B} \ln(A + Bx)\Big|_{t=0}^{t} = t$$

$$\Rightarrow \frac{1}{B} \ln(A + Bx(t)) - \frac{1}{B} \ln(A + B \underbrace{x(0)}_{=0}) = \frac{1}{B} \ln(A + Bx(t)) - \frac{1}{B} \ln(A) = \frac{1}{B} \ln\left(\frac{A + Bx(t)}{A}\right) = t$$

$$\ln\left(\frac{A + Bx(t)}{A}\right) = B \cdot t \Rightarrow \frac{A + Bx(t)}{A} = e^{B \cdot t} \Rightarrow A + B x(t) = A e^{B \cdot t} \Rightarrow \boxed{x(t) = \frac{1}{B}\left(A e^{B \cdot t} - A\right)}$$