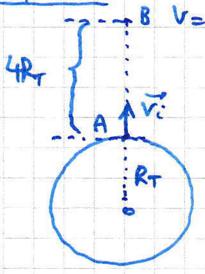
$\boxed{C8/E47}$



· En A, $E_{méc} = \frac{1}{2} m v_i^2 - GM\frac{m}{R_T}$

· En B, $E_{méc} = 0 - GM\frac{m}{5R_T}$

Il y a conservation de l'énergie entre A et B, car $W_{\Sigma F_{NC}} = 0$

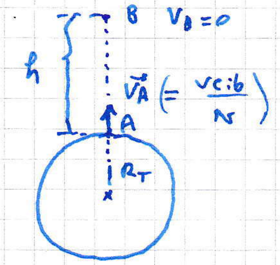
$$\Rightarrow \frac{1}{2}\cancel{m} v_i^2 - GM\frac{\cancel{m}}{R_T} = -GM\frac{\cancel{m}}{5R_T} \quad ; \quad v_i^2 = 2\left(\frac{GM}{R_T} - \frac{GM}{5R_T}\right) = \frac{8GM}{5R_T}$$

$$\Rightarrow v_i = \sqrt{\frac{8GM}{5R_T}} \approx \underline{10'000 \text{ m/s}}$$

$\boxed{C8/E49}$ $\quad V_{lib} = \sqrt{2G\frac{M}{r}} = \sqrt{2\cdot G\cdot 5000} \cong \underline{8{,}17\cdot 10^{-4} \text{ m/s}}$

Ici, $M = 5000$ kg ; $r = 1$ m

$\boxed{C8/E46}$



· En A, $E_{méc} = \frac{1}{2} m \frac{V_{lib}^2}{N^2} - GM\frac{m}{R_T}$

· En B, $E_{méc} = 0 - GM\frac{m}{R_T+h}$

il y a conservation de l'énergie entre A et B, car $W_{\Sigma F_{NC}} = 0$

$$\Rightarrow \frac{1}{2}\cancel{m}\frac{V_{lib}^2}{N^2} - GM\frac{\cancel{m}}{R_T} = -GM\frac{\cancel{m}}{R_T+h}$$

$$\Rightarrow \frac{V_{lib}^2}{2N^2GM} - \frac{1}{R_T} = -\frac{1}{R_T+h} \qquad \text{or, } V_{lib}^2 = 2G\frac{M}{R_T}$$

$$\Rightarrow \frac{\cancel{2GM}}{\cancel{2}N^2\cancel{GM}R_T} - \frac{1}{R_T} = -\frac{1}{R_T+h} \; ; \; \frac{1}{N^2R_T} - \frac{1}{R_T} = -\frac{1}{R_T+h} \; ; \; \frac{1}{R_T+h} = \frac{1}{R_T} - \frac{1}{N^2R_T} = \frac{1}{R_T}\left(1-\frac{1}{N^2}\right)$$

$$\frac{1}{R_T+h} = \frac{1}{R_T}\cdot\left(\frac{N^2-1}{N^2}\right) \; ; \; R_T + h = R_T\cdot\frac{N^2}{N^2-1} \quad \Rightarrow h = R_T\left(\frac{N^2}{N^2-1} - 1\right) = R_T\cdot\left(\frac{N^2-N^2+1}{N^2-1}\right)$$

$$\Rightarrow \underline{\underline{h = R_T\cdot\frac{1}{N^2-1}}}$$

$\boxed{C8/E55}$ i). Calcul de R géostationnaire :

Géostationnaire = période de révolution de 24 heures

$$\Rightarrow T = \frac{2\pi R}{V_{orb}} = 24\cdot 3600 \text{ et } V_{orb} = \sqrt{\frac{GM}{R}} \approx 3080 \text{ m/s} \quad \Rightarrow T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} \; ; \; T^2 = \frac{4\pi^2R^3}{GM}$$

$$\Rightarrow R = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} \approx 4{,}2\cdot 10^7 \text{ m}$$

ii). Calcul du travail nécessaire pour "monter" le satellite

$$W_{\Sigma \vec{F}_{ext\,NC}} = \Delta E_{cin} + \Delta E_{pot} = E_{cin\,f} - E_{cin\,i} + E_{pot\,f} - E_{pot\,i} = \frac{1}{2}mV_{orb}^2 - 0 + GM\frac{m}{R} + GM\frac{m}{R_T}$$

$E_{cin\,i} \approx 0$ (on néglige !)

$$= \frac{1}{2}mV_{orb}^2 - GMm\left(\frac{1}{R} - \frac{1}{R_T}\right) \cong \underline{1{,}22\cdot 10^{11} \text{ J}}$$

On remarque ici que au maximum, $E_{cin\,i} \cong \frac{1}{2}m\cdot 450^2 \doteq 2{,}1\cdot 10^8$ J

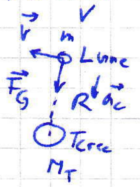
alors que $GM\frac{m}{R} \approx 1{,}3\cdot 10^{11}$ J

et $GM\frac{m}{R_T} \approx 2\cdot 10^{10}$ J $\quad \Rightarrow$ on néglige $E_{cin\,i}$!

et $\frac{1}{2}mV_{orb}^2 \approx 9{,}96\cdot 10^9$ J

C6/E46

a). $T=\frac{2\pi R}{V} \Rightarrow V=\frac{2\pi R}{T}$ (cinématique MCU)



$\underline{\Sigma\vec F = m\vec a}$ (Newton)

Projeté

$\Rightarrow F_G = m a_c$ (sur un axe Lune-Terre)

$\Rightarrow G\cancel{m}\frac{M_T}{R^2}=\cancel{m}\frac{V^2}{R}$; $G\frac{M_T}{R}=V^2$; $M_T=\frac{RV^2}{G}=\frac{R}{G}\cdot\frac{4\pi^2R^2}{T^2}=\frac{4\pi^2}{G}\cdot\frac{R^3}{T^2}=\frac{4\pi^2}{G}\cdot\frac{(3,84.10^8)^3}{(27,3.24.3600)^2}$

$\cong \underline{\underline{6.10^{24}\ kg}}$

b). Même raisonnement, mais : $T=365,25.24.3600$

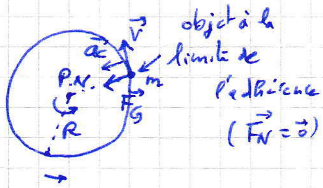
$R=1,5.10^{11}\ m$

$\Rightarrow M_S=\frac{4\pi^2}{G}\cdot\frac{(1,5.10^{11})^3}{(365,25.24.3600)^2}\cong \underline{\underline{2.10^{30}}}\ kg$

C6/E47

En utilisant ce qui a été fait lors de l'exercice E47 : $\frac{M_J}{M_T}=\frac{\cancel{\frac{4\pi^2}{G}}\cdot\frac{(6,7.10^5)^3}{(3,5)^2}}{\cancel{\frac{4\pi^2}{G}}\cdot\frac{(3,84.10^5)^3}{(27,3)^2}}=\frac{(6,7)^3}{(3,84)^3}\cdot\frac{(27,3)^2}{(3,5)^2}\cong\underline{\underline{323,16}}$

C6/E49



• Pour l'objet de masse m, $\Sigma\vec F = m\vec a_c$ (Newton)

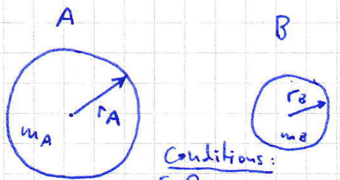
$\Rightarrow F_G = m a_c = m\frac{V^2}{R}$ (sur un axe radial)

$\Rightarrow G\cancel{m}\frac{M}{R^2}=\cancel{m}\frac{V^2}{R}$; $M=\frac{RV^2}{G}$

• Calcul de V (cinématique) $T=\frac{2\pi R}{V}=1 \Rightarrow V=2\pi R$

$\Rightarrow M=\frac{R}{G}\cdot 4\pi^2R^2=\frac{4\pi^2}{G}R^3\cong\underline{\underline{4,7.10^{24}\ kg}}$ (!)

C6/E54



Conditions :

$\begin{cases} r_A = 2r_B \\ \rho_A=\frac{\rho_B}{2}\end{cases}$

• Orbites "de basse altitude", de rayons r_A et r_B respectivement

• $T_A=\frac{2\pi r_A}{v_A}$; $T_B=\frac{2\pi r_B}{v_B}$ "Comparer" = chercher le rapport $\frac{T_A}{T_B}$

• $v_A=\sqrt{G\cdot\frac{m_A}{r_A}}$; $v_B=\sqrt{G\cdot\frac{m_B}{r_B}}$

• $\rho_A=\frac{m_A}{\frac{4}{3}\pi r_A^3}$; $\rho_B=\frac{m_B}{\frac{4}{3}\pi r_B^3}$

$$\frac{T_A^2}{T_B^2}=\frac{\cancel{4\pi^2}r_A^2}{v_A^2}\cdot\frac{v_B^2}{\cancel{4\pi^2}r_B^2}=\left(\frac{r_A}{r_B}\right)^2\cdot\frac{\cancel{G}\frac{m_B}{r_B}}{\cancel{G}\frac{m_A}{r_A}}=\left(\frac{r_A}{r_B}\right)^2\cdot\frac{m_B}{m_A}\cdot\frac{r_A}{r_B}=\left(\frac{r_A}{r_B}\right)^3\cdot\frac{m_B}{m_A}=\cancel{\left(\frac{r_A}{r_B}\right)^3}\cdot\frac{\cancel{\frac{4}{3}\pi}\rho_B \cancel{r_B^3}}{\cancel{\frac{4}{3}\pi}\rho_A \cancel{r_A^3}}=\frac{\rho_B}{\rho_A}$$

$\Rightarrow \frac{T_A^2}{T_B^2}=\frac{\rho_B}{\rho_A}=\frac{\rho_B}{\rho_B/2}=2 \qquad \Rightarrow \underline{\underline{\frac{T_A}{T_B}=\sqrt2}}$

C8/E68 On utilise le théorème de l'énergie cinétique. On n'a pas besoin de connaître le détail de $\sum \vec{F}_{ext}$.

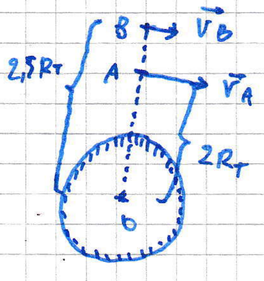
$\simeq 0$ (rotation de la Terre)

$$W_{\sum \vec{F}_{ext\,NC}} = \Delta E_{cin} + \Delta E_{pot} = E_{cin\,f} - E_{cin\,i} + E_{pot\,f} - E_{pot\,i} = \frac{1}{2} m v_{orb}^2 - GM\frac{m}{R} + GM\frac{m}{R_T}$$

$$= \frac{1}{2} m \frac{GM}{R} - GM\frac{m}{R} + GM\frac{m}{R_T} = -\frac{1}{2} m \frac{GM}{R} + GM\frac{m}{R_T} = GMm\left(\frac{1}{R_T} - \frac{1}{2R}\right) \cong \underline{\underline{1{,}9\cdot 10^9\ \text{J}}}$$

$$\begin{cases} R_T = 6{,}37\cdot 10^6\ \text{m} \\ R = (6{,}37\cdot 10^6 + 0{,}0039\cdot 10^6)\ \text{m} \end{cases}$$

C8/E70



Travail requis : $W_{\sum \vec{F}_{ext\,NC}} = \Delta E_{cin} + \Delta E_{pot}$ (thm. de l'énergie cinétique)

$$W = E_{cin\,f} - E_{cin\,i} + E_{pot\,f} - E_{pot\,i} = \frac{1}{2} m v_{orb\,B}^2 - \frac{1}{2} m v_{orb\,A}^2 - GM\frac{m}{R_B} + GM\frac{m}{R_A}$$

$$= \frac{1}{2} m G\frac{M}{R_B} - \frac{1}{2} m G\frac{M}{R_A} - GM\frac{m}{R_B} + GM\frac{m}{R_A} = \frac{1}{2} G\frac{mM}{R_A} - \frac{1}{2} G\frac{mM}{R_B}$$

$v_{orb} = \sqrt{\frac{GM}{r}}$

$$= \frac{1}{2} GmM\left(\frac{1}{R_A} - \frac{1}{R_B}\right)$$

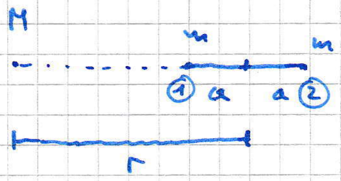
$$= \frac{1}{2} GmM\cdot\left(\frac{1}{2R_T} - \frac{1}{2{,}5R_T}\right)$$

$$= \frac{GmM}{2R_T}\cdot\left(\frac{1}{2} - \frac{1}{2{,}5}\right) = \frac{GmM}{20R_T}$$

$$\cong \underline{\underline{7{,}2\cdot 10^8\ \text{J}}}$$

C13/E14

$r \gg a$: on considère M et m ponctuelles !



Force de gravité :

$$F_1 = G\cdot M\cdot\frac{m}{(r+a)^2}$$

$$F_2 = G\cdot M\cdot\frac{m}{(r-a)^2}$$

$$\Delta F = F_1 - F_2 = GMm\cdot\left(\frac{1}{(r+a)^2} - \frac{1}{(r-a)^2}\right) = GMm\cdot\left(\frac{(r-a)^2-(r+a)^2}{(r+a)^2(r-a)^2}\right)$$

$$= GMm\cdot\left[\frac{r^2+a^2-2ar-r^2-a^2-2ar}{(r^2+a^2+2ar)(r^2+a^2-2ar)}\right] = GMm\cdot\frac{-4ar}{r^2\Big(1+\underbrace{\left(\frac{a}{r}\right)^2}_{\to 0}+\underbrace{2\left(\frac{a}{r}\right)}_{\to 0}\Big)\,r^2\Big(1+\underbrace{\left(\frac{a}{r}\right)^2}_{\to 0}-\underbrace{2\left(\frac{a}{r}\right)}_{\to 0}\Big)}$$

$$= -4GMma\frac{r}{r^4} = \underline{\underline{-4GMma\cdot\frac{1}{r^3}}}$$

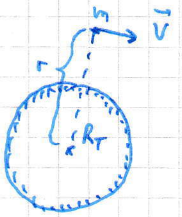
$\boxed{C13/P1}$

"Basse altitude"

$$\cdot\, T = \frac{2\pi R}{v} = \frac{2\pi R}{\sqrt{G\frac{M}{R}}} = \frac{2\pi R}{\sqrt{\frac{G\cdot\frac{4}{3}\rho\pi R^3}{R}}} = \frac{2\pi R}{\sqrt{G\cdot\frac{4}{3}\rho\pi R^2}} = \frac{2\pi \cancel{R}}{\cancel{R}\sqrt{G\,4/3\,\rho\pi}} = \underline{\underline{\frac{2\pi}{\sqrt{G\cdot\frac{4}{3}\rho\pi}}}}$$

$$\cdot\, \rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} \Rightarrow M = \frac{4}{3}\rho\pi R^3$$

$\boxed{C8/E69}$



a). $E_{cin} = \frac{1}{2} m v^2 = 75\cdot(6000)^2 = \underline{\underline{2{,}7\cdot 10^9\,J}}$

b). $E_{pot} = -GMm\frac{1}{r} = -G\cdot M\cdot m\cdot\frac{6000^2}{GM} = -m\cdot 6000^2 = -2E_{cin} = \underline{\underline{-5{,}4\cdot 10^9\,J}}$

Comment trouver r ?

$$v_{orb} = \sqrt{\frac{GM}{r}} = 6000$$

$$\frac{GM}{r} = 6000^2$$

$$\Rightarrow r = \frac{GM}{6000^2}$$

c). $E_{méc} = E_{cin} + E_{pot} = \underline{\underline{-2{,}7\cdot 10^9\,J}}$ état lié !

d). $E_{minimale} = \underline{\underline{2{,}7\cdot 10^9\,J}}$

car $W_{\Sigma F_{ext}NC} = \Delta E_{cin} + \Delta E_{pot} = E_{cin\,i} - \underbrace{E_{cin\,f}}_{=0} + E_{pot\,i} - \underbrace{E_{pot\,f}}_{=0} = E_{cin\,i} + E_{pot\,i}$

$v \to 0$ $\quad$ $r \to \infty$
